Spectral gravity forward modelling of continuous 3D variable density contrasts using an arbitrary integration radius

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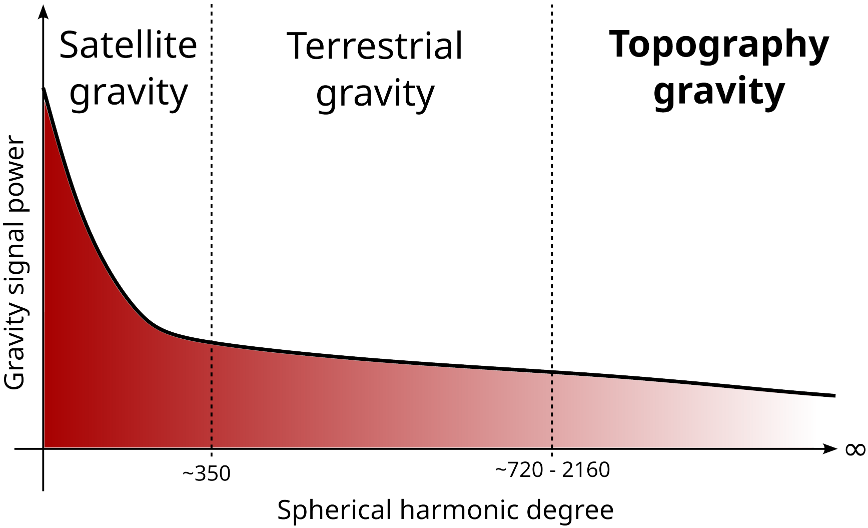
Motivation


Satellite gravity
Terrestrial gravity
Topography gravity
Gravity signal power
~350
~720 - 2160
∞
Spherical harmonic degree

Method

- Topographic potential:

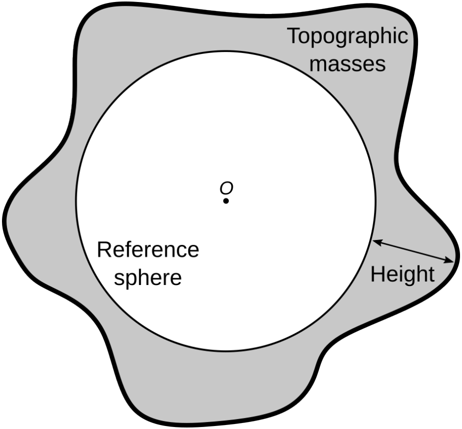
$$V(r,\Omega) = G \iint\limits_{\Omega'} \int\limits_{r'=R}^{R+H(\Omega')} \frac{\rho(r',\Omega')}{\ell(r,\psi,r')} \times (r')^2 \, \mathrm{d}r' \, \mathrm{d}\Omega'$$

- 3D density:

$$\rho(r',\Omega') = \sum_{i=0}^{\infty} \rho_i(\Omega') \, (r')^i$$

with

$$\rho_i(\Omega') = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \bar{\rho}_{nm}^{(i)} \, \bar{Y}_{nm}(\Omega')$$



New Spectral Technique (Global Variant)

- The new spectral technique:

$$\boxed{V(r,\Omega) = \frac{GM}{R} \sum_{n=0}^{\infty} \left(\frac{R}{r}\right)^{n+1} \sum_{m=-n}^{n} \bar{V}_{nm}\, \bar{Y}_{nm}(\Omega),}$$

where

$$\bar{V}_{nm} = \frac{2\pi\, R^3}{M} \sum_{i=0}^{\infty} \sum_{p=1}^{n+i+3} S_{npi}\, \overline{\mathrm{H}\rho}_{nm}^{(pi)},$$

$$S_{npi} = \frac{2}{2n+1}\, \frac{1}{n+i+3} \binom{n+i+3}{p},$$

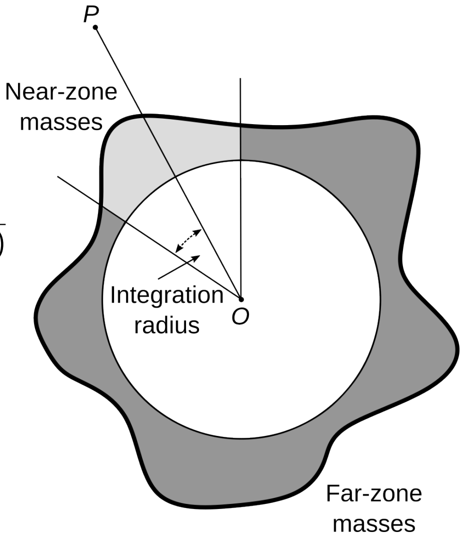
$$\overline{\mathrm{H}\rho}_{nm}^{(pi)} = \frac{R^i}{4\pi} \iint\limits_{\Omega'} \left(\frac{H(\Omega')}{R}\right)^p \rho_i(\Omega')\, \bar{Y}_{nm}(\Omega')\, \mathrm{d}\Omega'.$$

New Spectral Technique (Cap-modified Variant)

Near- and far-zone effects:

$$V^{j}(r,\Omega,\psi_0) = G \iint\limits_{\Omega'_j} \int\limits_{r'=R}^{R+H(\Omega')} \frac{\rho(r',\Omega')}{\ell(r,\psi,r')} \times (r')^2 \,\mathrm{d}r' \,\mathrm{d}\Omega'$$

with $j = \{\text{'Near'}, \text{'Far'}\}$.



New Spectral Technique (Cap-modified Variant)

- The new spectral technique:

$$\boxed{V^{j}(r,\Omega,\psi_0) = \frac{GM}{R}\sum_{n=0}^{N}\sum_{m=-n}^{n}\bar{V}_{nm}^{0,0,j}(r,\psi_0,R)\,\bar{Y}_{nm}(\Omega),}$$

$$\bar{V}_{nm}^{uvj}(r,\psi_0,R) = (-1)^{v}\,\frac{2\pi\,R^{3+u}}{M}\sum_{p=1}^{P}\sum_{i=0}^{I}Q_{npi}^{uvj}(r,\psi_0,R)\,\overline{\mathrm{H}\rho}_{nm}^{(pi)}.$$

- Extended up to the full gravitational tensor and all its radial derivatives

Implementation in CHarm

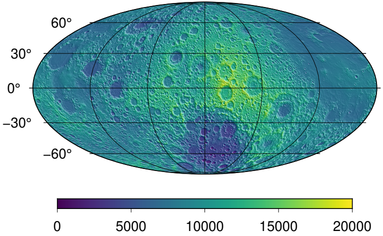
- **C/Python library** for high-degree spherical harmonic transforms
- https://charmlib.org
- Free software
- Arbitrary-degree transforms
- Single, double and quadruple precision
- OpenMP and SIMD parallelization
- FFTs by FFTW
- Multiple-precision floating-point computations by MPFR
- Easy installation in Python:

```
$ pip install pyharm
```

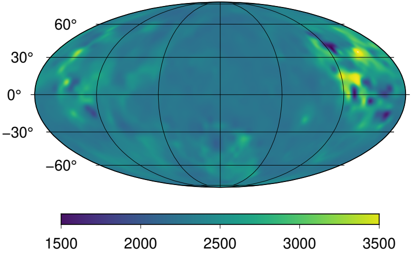
CHarm's logo

Experiment – Lunar Topographic Masses

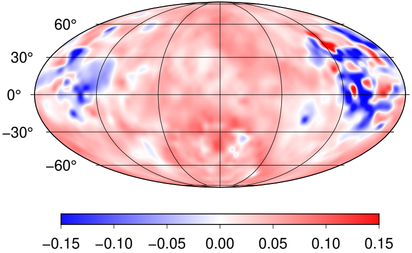
Lunar topography (m) up to degree 1080 (MoonTopo2600p [Wieczorek, 2015])



Surface density
$(\mathrm{kg\ m}^{-3}$; [Goossens et al., 2020])

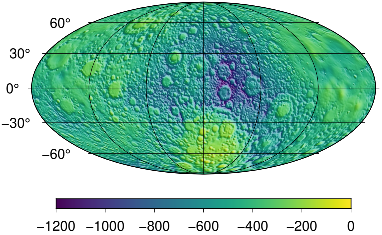


Gradient of the surface density
$(\mathrm{kg\ m}^{-4}$; [Goossens et al., 2020])

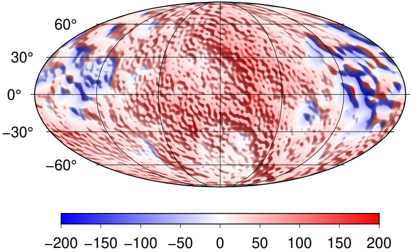


Experiment – Radial Component of the Gravitational Vector

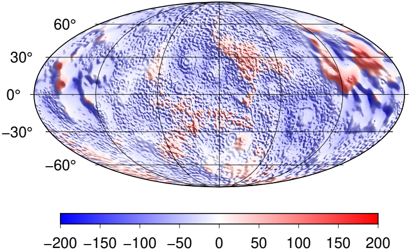
$V^{\mathrm{Near,z}}$ from 3D density at satellite altitude
(mGal; max. degree 3240, integration radius 1°)



3D density vs. constant density (mGal)



3D density vs. lateral density (mGal)



Conclusions

- Spectral technique generalized to 3D-variable densities and arbitrary integration radius
- Implemented in CHarm (https://charmlib.org)
- Far-zone gravitational effects likely convergent even on the topography [Bucha and Kuhn, 2020]
- High-resolution **surface** gravitational maps based on 3D density

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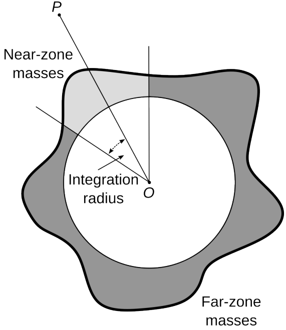

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Thank you for Your Attention!

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RECOVERY AND RESILIENCE PLAN

References

Bucha, B. and Kuhn, M. (2020).
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Goossens, S., Sabaka, T. J., Wieczorek, M. A., Neumann, G. A., Mazarico, E., Lemoine, F. G., Nicholas, J. B., Smith, D. E., and Zuber, M. T. (2020).
High-resolution gravity field models from GRAIL data and implications for models of the density structure of the Moon's crust.
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Wieczorek, M. A. (2015).
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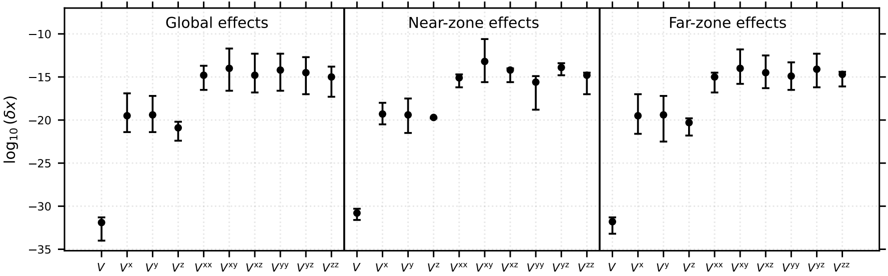
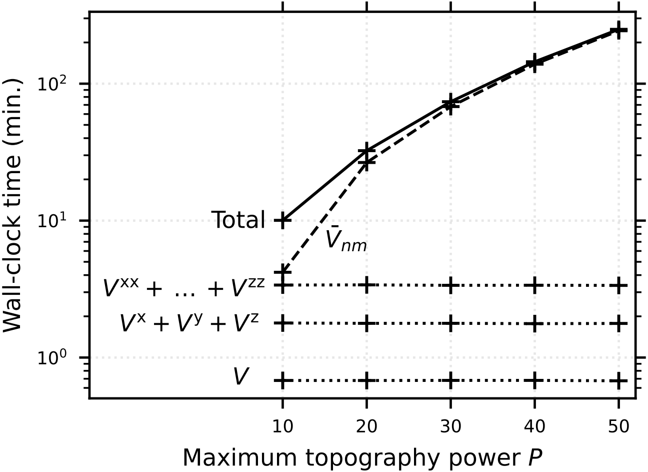
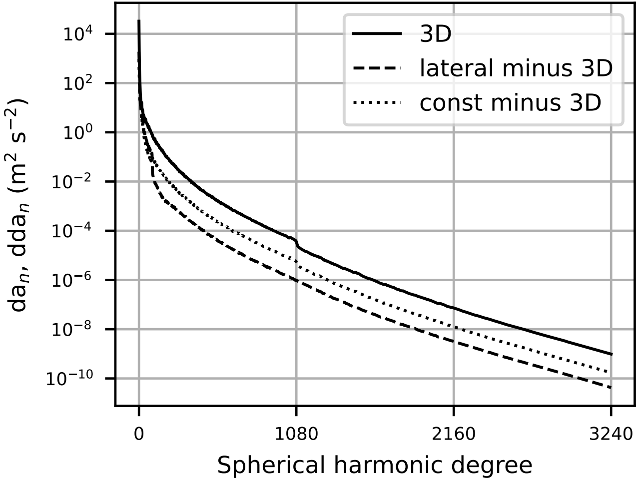
Backup slides



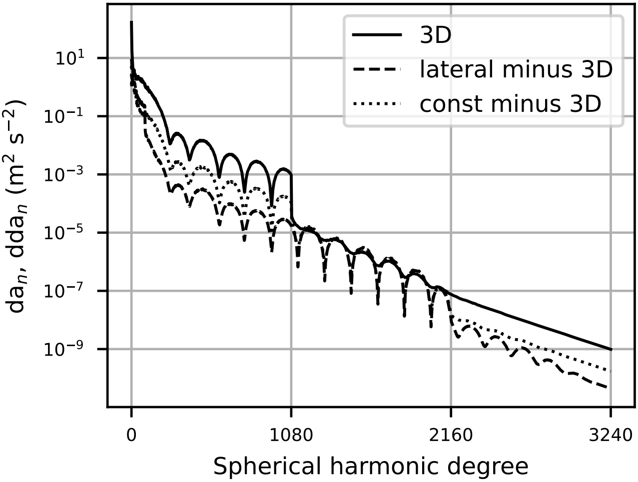
Base-10 logarithm of relative errors of the new spectral method. The filled circles show the mean values of $\log_{10}(\delta x)$ *and the error bars indicate the minimum and maximum values. Both the spectral and the reference technique are implemented in quadruple precision.*



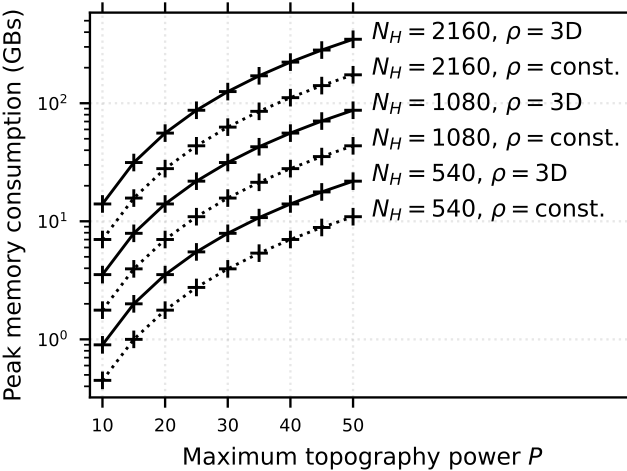
Wall-clock time it took CHarm to perform global spectral gravity forward modelling for fixed $N_H = 1080$, $N = 10{,}800$, $I = 1$, $N_{\rho_0} = N_{\rho_1} = 90$ and varying P. The time to evaluate $\bar{V}_{nm}$ is shown by the dashed curve, the synthesis times of gravitational quantities are shown by the dotted curves (the gravitational potential V, the full gravitational vector $V^{\mathrm{x}} + V^{\mathrm{y}} + V^{\mathrm{z}}$ and the full gravitational tensor $V^{\mathrm{xx}} + \cdots + V^{\mathrm{zz}}$) and the solid curve represents their sum.



Spectrum of the gravitational potential implied by a 3D density distribution (degree amplitudes) and its differences with respect to constant- and lateral-density-based potentials (difference degree amplitudes). All spectra refer to the Brillouin sphere of the radius 1,750,000 m *passing outside of all gravitating masses.*



Same as previous figure but for near-zone gravitational effects $V^{z,In}$ with the integration radius of $\psi_0 = 1^\circ$.



Theoretical peak memory consumption of the global variant. Shown is the amount of memory required by CHarm to evaluate the potential coefficients in double precision without aliasing for a varying N_H and fixed $N_{\rho_0} = N_{\rho_1} = 90$, $N = 10{,}800$ (solid curves). For a comparison, the dotted lines show the peak memory consumption when using a constant mass density in CHarm.